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Technical Paper

Mathematical Models in Psychology for Stochastic Description of Human Feelings

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Abstract: We reviewed the application of mathematical analysis in psychology research, especially on human feelings. As mathematical analysis is widely used for modeling, it also has been applied to study human behavior. There are problems in applying mathematical analysis to psychology research due to the inherent subjectivity with multi-facets, intricacy, and dependence on context and memory of human feelings. Therefore, non-linear mathematical models and artificial intelligence engines are demanded to mimic human intuition for the better performance of models and, further, personalizing models accommodating individual differences. For this, adaptive learning methods need to be employed. Along with such technological considerations, diverse and reliable datasets are also mandatory to construct appropriate mathematical models. Human feelings are intricate and multifaceted, and cannot be easily expressed in mathematics. Therefore, it is necessary to develop mathematical analysis methods to understand and assess human feelings more precisely. With the development of computational technology and data-gathering techniques, existing problems can be solved, and data including sentiment analysis results and facial expressions with time-dependent components can be available. It is expected to have personalized models based on individual differences soon owing to the rapid development of mathematical analysis techniques. Such models to assess and predict human feelings can be used for diverse applications including psychology, marketing, education, and others.

Keywords: Mathematical Analysis, Modeling, Human Feelings, Emotions, Quantitive Assessment, Artificial Intelligence, Non-linear Model

1. Introduction

Mathematical analysis mainly deals with continuous functions, limits, and related theories based on concepts including differentiation, integration, measure, infinite sequences, series, and analytic functions which are applied within the framework of real and complex numbers and functions (Hewitt and Stromberg, 1965). Mathematical analysis is rooted in calculus to apply elementary concepts and techniques for analysis and plays a vital role in the following research areas but is not limited to (Foster, 2023; Conocimiento, 2020; Richard, 2023)

1. Formulation of theories: Mathematics enables scientists to formulate theories that describe natural phenomena, physical processes, and complex systems. These theories involve mathematical models expressed through equations.
2. Quantifiable measurement: Mathematical analysis provides a framework for quantifying and measuring various phenomena. Researchers can express quantitive measurements precisely: for example, tracking the spread of diseases or analyzing physical properties.
3. Statistical analysis: Statistics is used to draw significant conclusions from data. Various statistical methods are used to analyze experimental results, validate hypotheses, and make predictions.
4. Logical deduction: Mathematics relies on logic and deduction. Scientists use mathematical reasoning to prove theorems, explore relationships between variables, and draw conclusions.

Mathematical analysis is a valuable tool for modeling, measuring, inferencing, and logical reasoning (Ohue et al., 2024). Advancements in mathematical modeling and problem-solving have contributed significantly to developing technologies related to computer vision, drug discovery, and materials science recently (Pérez-Escobar, 2023).

In psychology, mathematical analysis has been widely used mainly to unravel human behavior. While psychology delves into the complexities of the human mind, mathematics is used to understand and predict cognitive processes in it. Statistical analyses and computational models are the most often used means of mathematical analysis to understand the emotions, cognition, and decision-making of human (Foster, 2024). Psychology mainly relies on qualitative and theoretical methods and interpretations. However, the emergence of quantitative psychology based on mathematics allows for the understanding of perception, memory, attention, problem-solving, and decision-making (Richardson and Suinn, 1972).

As mathematical analysis is widely used in psychological studies, researchers have applied it to study human feelings. Human feelings are intricate and multifaceted experiences. Researchers have delved into how to understand emotional phenomena and their significance in lives. Human feelings are related to emotional functioning and the underlying developmental mechanisms. There are still many questions to be answered including the definition of emotion, the process of emotional changes, emotional regulation in different social environments, and cultural assessments (Pollak et al., 2019). As human feelings are intricate from joy to sorrow, they cannot be simply defined as reactions to an event or situation. Also, human feelings cannot be expressed on a numerical scale as there are no units of measurement for feelings. However, researchers use a numerical scale to express them to investigate their socioeconomic influences. Relationships between factors affecting human feelings and a measured scale of human feelings have been found as respondents managed to choose numerical scores systematically (Kaiser and Oswald, 2022). This enables quantitive research as shown in Table 1. The results show that human beings operationalize an integer scale for feelings, which leads to inherently cross-disciplinary studies. Table 1 shows how mathematical analysis has been applied in various psychological research.

Table 1. Applications of mathematical analyses in diverse research areas of psychology.

Topic	Mathematical Analysis	Important Findings	References
Mood Disorders	Statistical modeling, pattern recognition algorithms	Mathematical correlations in mood data and patterns associated with depression, anxiety, and bipolar disorders	Anmol (2020)
Cognitive Dysfunctions	Quantitative methods, data-driven assessments	Understanding of cognitive impairments, aiding diagnosis, and treatment strategies	
Behavioral Anomalies	Pattern recognition, behavioral mathematics	Mathematical models to predict and analyze behavioral deviations for personalized interventions	
Memory and Attention	Computational models, neural network analysis	Mathematical frameworks to explain memory processes, attentional shifts, and cognitive load	Foster (2024)
Decision Making	Game theory, utility functions	Mathematical decision models for optimal choices, biases, and risk-taking behavior	Cavagrano et al. (2013) Cooper et al. (2023)
Psychometrics	Factor analysis, item response theory, latent variable modeling	Psychological constructs, intelligence tests, personality assessments, and surveys to quantify mental attributes	Ryser (2023) Edelsbrunner, and Dablander (2019)
Unveiling patterns	Regression, ANOVA, and correlation, Bayesian statistics, bootstrapping, and mixed-effects models	Identifying associations of psychological phenomena	Veenman et al. (2023) Bakdash and Marusich (2017) Blanca et al. (2018)
Emotions as mathematical entities	Quantifying emotional intensity and identifying qualitative patterns	Mathematical description of personal experiences	Bieleke et al. (2023)

Therefore, in this study, we aim to review how to develop and apply mathematical analysis methods to understand human feelings better and even assess human emotions further. With the development of computational technology and data-gathering techniques, the review results and suggestions serve as the basis for further assessment of human feelings and the development of mathematical methods for it.

2. Human Feelings and Emotions in Psychological Aspect

Psychology is the academic field of studying the human mind and its functions that affect human behavior (Oxford Languages, 2024). Mind is defined as an element of an individual that feels, thinks, and reasons (Merriam-Webster, 2024). Emotions arise depending on human minds, specifically human thoughts (Antonio, 2022). As psychology is interpreted in diverse aspects, multiple approaches have been proposed in psychology. In behavioral psychology, human behaviors are regarded as responses to stimuli. Cognitive psychology studies the human perception process in the mind. Psychodynamic psychology describes human behaviors through an individual's experiences. In biological psychology, all emotions and feelings are thought to be produced in a biological process. In detail, for sensed environmental stimulus, the limbic system of the brain manages diverse responses (Jack, 2024). The limbic system identifies emotions that an individual feels and releases neurochemicals depending on the emotion. These neurochemicals result in changes in posture, facial expression, and unconscious body activities, such as heart rate and blood pressure. Lastly, humanistic psychology emphasizes a unique entity and the perception of each individual. Except for humanistic psychology, the four aspects of psychology have developed reciprocally with math, science, and technology over the past few decades (William, 2015). The causality and relationship between the stimulus and responses are explicit with accumulated data. Human thoughts and cognition are schematized using computers. The relationship between childhood experiences and future behaviors has been compiled using computers, for example. The process in the brain has been researched focusing on connections and relationships at each stage.

3. Mathematical Analysis of Human Feelings

Human feelings are influenced by emotions but are generated from thoughts. With the physical experience of an emotion, feelings are formed as mental sensations. Even the same sensations can lead to different feelings such as awkwardness, excitement, or intimidation depending on mental interpretation. Understanding and assessing human feelings helps to explore self-awareness and empathy (ThinkPsych, 2022; Betterhelp, 2024). The emotional phenomenon triggering human feelings has been researched in Psychology, Psychiatry, Neuroscience, and Biochemistry. However, since the perception of emotions is not directly detectable nor recordable, physics and mathematics have not been so far used to describe feelings and emotions. Since Elahi hypothesized that the human soul and its psyche may be assessed using electromagnetic waves in conscious and unconscious manners, mathematical methods for the description of feelings and emotions have been developed. The measured intensity and inferred nature of feelings of the emotional phenomenon provide a scientific approach to understanding the nature of the mind and the physiological manifestation of emotions.

Carbonaro and Giordano (2005) studied how to mathematically describe human feelings. They took a simple approach to describe the psychological state using a single variable as a measure of feelings and modeled reciprocal feelings in social relations. The proposed model employed nonlinear integro-differential stochastic equations for probabilistic forecasts of feelings between two related individuals. The solution was expressed as probability density functions which were validated with external and internal parameters. Based on the result, Carbonaro and Giordano (2005) proposed the stochastic mathematical description of human feelings by defining a state of feelings of system Σ as follows.

$$\mathbf{U} \equiv \begin{pmatrix} 0 & u_{12} & u_{13} & \cdots & u_{1n} \\ u_{21} & 0 & u_{23} & \cdots & u_{2n} \\ u_{31} & u_{32} & 0 & \cdots & u_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & u_{n3} & \cdots & 0 \end{pmatrix} \tag{1}$$

Here, the probability function is

$$\pi_{ij}: x \in \mathbb{R} \longrightarrow \pi_{ij}(x) \in [0, +\infty) \tag{2}$$

Then, the probability P is expressed as

$$\mathbf{P}\left(a \le u_{ij} \le b\right) \equiv \mathbf{P}_{ij}(a,b) = \int_a^b \pi_{ij}(x)dx \tag{3}$$

Then, the individual feeling of *ii* towards *ij* is expressed as the expected value as follows.

$$E\left(u_{ij}\right) = \int_{-\infty}^{+\infty} x\pi_{ij}(x)dx \tag{4}$$

Mathematical modeling has been used to describe emotions and their subsequent feelings and behavior in more advanced way by Ambrosio (2020). He assumed that electromagnetic waves could be used to measure the unconscious and conscious levels of the human mind. The electromagnetic wave was manifested by its spectrum and intensity in a main frequency, Thus, using wave equations, the evolution of the psychic state over time was described by the following equation.

$$\partial\frac{\partial}{\partial t}\Psi = \int_{\Omega} F(\Psi)dx \tag{5}$$

where Ω represents the domain of the mind and F the reaction of the self to external and internal events.

Assuming that the human psychic state results from the summation of distinct single-frequency wave components, the electromagnetic wave can be decomposed into the spectrum of individual sinusoidal components. Thus, the psychic state $\Psi(t, z)$ of an individual at time t and space z is described as Equation (2).

$$\Psi(t,z) = \int_{\xi \in R} P_{\xi}(t)\, \varphi_{\xi}(z)d\xi \tag{6}$$

where $(\phi\xi)\xi \in R$ represents a basis of fundamental frequencies of emotions.

Equation (2) is similar to the Fourier Transform and describes a continuous model. Based on the equation, Ambrosio (2020) described anger as Equation (3).

$$f_{\sigma}(x,y) = \frac{1}{2\sigma\pi}\exp\left(-\frac{x^2+y^2}{2\sigma}\right) \tag{7}$$

Recently, owing to the development of artificial intelligence technology, researchers have applied it to various studies including psychology. Katharine (2023) summarized major findings in various fields using AI engines and claimed that the effectiveness of AI engines varies by research area and collected data. Using the concept of social intelligence, Sufyan et al. (2024) found that AI language models show rapid learning ability in understanding emotions and social behavior and suggested exploring how to use AI-powered engines for psychotherapy.

The University of Jyväskylä, Finland (2024) developed a computer model to interpret and understand human emotions. Based on the principles of mathematical psychology, they developed the interface between users and AI systems, making interactions intuitive and responsive. To solve the problem of the lack of understanding of human emotions, their model was trained to predict emotions by simulating the cognitive evaluation process and satisfactorily predicted happiness, boredom, irritation, rage, despair, and anxiety. They applied the sequential decision-making model, appraisal calculation, and classifier to augment the model. They employed a Markov decision process (MDP) which is used to model stochastic decision-making problems. It is a tuple < S, A, T, R, γ >, where S is the set of states and A is the set of actions of the agent. The state transition function T(s, a, s′) presents the probability of transitioning state s ∈ S to s′ ∈ S with a ∈ A. The reward function R(s, a, s′) is used to measure the immediate reward the agent receives from state s to s′ in doing a. The discount factor γ reduces future rewards in the value of actions (Zhang et al., 2024). In their sequential decision-making model, the value function vπ(s) of a state s under a policy π is used to quantify the expected return.

$$v_{\pi}(s) = \mathbb{E}_{\pi}[G_t \mid S_t = s], \text{ for all } s \in S, \tag{8}$$

The value of an action a ∈ A in s ∈ S is defined as

$$q_{\pi}(s,a) = \mathbb{E}_{\pi}[G_t \mid S_t = s, A_t = a], \text{ for all } s \in S \text{ and } a \in A \tag{9}$$

where $G_t = \sum_{k=0}^{\infty} \gamma^k R_k$ is the expected discounted return, and $\mathbb{E}_{\pi}$ is the expected value of the policy.

In appraisal calculation, suddenness as a part of the assessment of an event, and the goal relevance and goal conduciveness of an event are expressed as

$$A_s = 1 - \frac{\hat{T}(s,a,s')}{\sum_{s'' \in S} \hat{T}(s,a,s'')} \tag{10}$$

$$A_{gr} = min(1, |\Delta|), \tag{11}$$

$$A_{gc} = min(max(\Delta, -1), 1) * 0.5 + 0.5 \tag{12}$$

where T is a world model, $\Delta(s,a) = \alpha |R(s,a) + \gamma max_{a'} q(s',a') - q(s,a)|$ is the TD error.
Finally, they augmented the model by using a simple moving window average as follows.

$$SMA(t) = \frac{x(t) + x(t-1) + \cdots + x(t-n)}{n+1} \tag{13}$$

where *x*(*t*) presents the prediction of an emotion *e* at time *t*, and *n* is the length of the window.

Quantitative psychological research relies on statistical methods and mathematical modeling to explore human behavior. Recent advancements such as Zhang et al.'s model enable a powerful tool for understanding emotions as well as the cognition and decision-making of humans. By bridging theory and data, novel approaches have been made for robust predictions of human feelings. While traditional hypothesis testing has been the foundation of psychological research, complex phenomena can be oversimplified as binary outcomes (significant or not) are obtained. Constructing mathematical models based on recent technologies, the nuances, relationships, and dynamic processes of human feelings can be explored. Mathematical psychology used for quantitative psychological research combines mathematics, cognitive science, and behavioral research to understand mental processes using decision theory, learning models, and neural networks. In the related studies, differential equations are used as fundamental tools. Dynamic systems such as learning curves, memory decay, and emotion dynamics can be presented and analyzed by solving the equations to understand psychological processes. Even though emotions are elusive and subjective, researchers have proposed mathematical modeling to assess and predict feelings and emotions. These models allow quantitative and qualitative assessments of emotional phenomena.

4. Problems and Solutions of Mathematical Analysis for Human Feelings

Even though advancements are witnessed in assessing and predicting human feelings using mathematical methods, there are problems and potential solutions with challenges and promising directions. Human feelings are inherently subjective and multifaceted, so traditional mathematical models may not be appropriate to quantify individual cultural, contextual, and personal variations. Human feelings also have intricate interactions in neural, physiological, and psychological processes. Therefore, linear models might oversimplify such complexity. Direct measurement cannot be used for human feelings so data must be collected by compiling self-reports and measuring physiological markers. Human feelings change over time, influenced by context and memory, and occur differently according to experience and emotions even in the same situation. People differ in how they experience and express emotions. Thus, consistent and reliable data are lacking. Therefore, it is mandatory to collaborate in a multidisciplinary way in psychology, neuroscience, and mathematics. This also leads to the problem of how to validate an established model and how to use it for psychotherapy. In addition, as the model collects and uses personal data, there is a chance of including biased or noisy data, which hinders the model from overlooking universal findings and recognizing dynamics (Ambrosio, 2020; ScienceDaily, 2024; Brody, 2022). There are also challenges in applying subjective emotional data to mathematical models. The quality of data is dependent on individual differences as emotions are inherently subjective, meaning that different individuals have a wide range of emotional responses to the same situation. Even when standardized questionnaires are used, people respond to questions with personal biases, which makes it difficult to quantify emotional data in mathematical models. Such a subjective nature of emotions may lead to inconsistent data collection (Barrett and Russell, 1999)

To overcome such challenges, it is necessary to adopt AI engines to mimic human intuition as more sophisticated methods. Person-specific models or dynamic systems approaches need to be developed and used for the complexity of emotional experience. This needs to develop nonlinear models to deal with the intricate dynamics of the data and account for subjectivity while maintaining generalizability. It is necessary to develop measures such as sentiment analysis results and facial expressions to use them for mathematical models. As the non-linear model using differential equations and Markov models is inevitable for modeling human feelings, time-dependent components need to be determined, especially for personalizing models based on individual differences. For this, adaptive learning methods need to be employed. Along with such technological considerations, diverse and reliable datasets must be obtained to foster interdisciplinary collaboration to enrich mathematical models from psychological perspectives. Especially, AI-powered tools must be developed regarding human expertise. However, AI systems require access to sensitive personal information which raises privacy concerns. Participants may not be fully aware of how their data is used. As AI models often require large amounts of data, ethical standards of informed consent must be obeyed (Mittelstadt et al., 2016)

5. Conclusions

Mathematical analysis is widely used in formulating theories, various measurements and predictions, statistical analyses, and logical deduction by using proven theorems and exploring relationships between variables. As it is mainly used for mathematical modeling and problem-solving, mathematical analysis is also used in psychology to quantify human behavior and understand and predict cognitive processes. Statistical analyses and computational models have been mainly used to understand the emotions, cognition, and decision-making of humans as qualitative and theoretical methods and interpretations are mostly employed. Quantitative psychology based on mathematics has widened the perception, memory, attention, problem-solving, and decision-making of human feelings. As human feelings are intricate and multifaceted, researchers have delved into how to understand emotional phenomena. However, human feelings cannot be easily expressed on a numerical scale. Therefore, mathematical analysis methods need to be developed to understand and assess human feelings. With the development of computational technology and data-gathering techniques, existing problems can be solved by quantifying cultural, contextual, and personal variations. As human feelings have intricate interactions in neural, physiological, and psychological processes, it is necessary to adopt AI technologies to develop nonlinear models. Along with this, various data including sentiment analysis results and facial expressions with time-dependent components are mandatory to model human feelings. It is expected to have personalized models based on individual differences, soon. The methodology of mathematical analysis has been more important than ever to assess and predict human feelings. The models constructed using it can be used for diverse applications including psychology, marketing, education, and others. However, the quality of data and ethical concerns must be addressed as individuals have a wide range of emotional responses to the same situation and sensitive personal information can be used by various models.

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